Global Biogeochemical Cycles

Supporting Information for

Partitioning N₂O Emissions within the US Corn Belt using an Inverse Modeling Approach

Z. Chen¹, T. J. Griffis¹, D. B. Millet¹, J. Wood¹, X. Lee², J. M. Baker¹,³, K. Xiao¹, P. A. Turner¹, M. Chen¹, J. Zobitz⁴, and K. C. Wells¹

¹ Department of Soil, Water, and Climate, University of Minnesota, Saint Paul, MN, USA
² School of Forestry and Environmental Studies, Yale University, New Haven, CT, USA
³ United States Department of Agriculture, Agricultural Research Service, Saint Paul, MN, USA
⁴ Department of Mathematics, Augsburg College, Minneapolis, MN, USA

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Text S1

Introduction

This supporting information provides additional details related to the Bayesian inversion.
Text S1. Solutions of Bayesian Inversion

\[ y = K\Gamma + \epsilon \quad \text{(S1)} \]

Where \( y \) is the observed minus background mixing ratios, the columns of \( K \) correspond to the simulated mixing ratios for each of the source types being optimized, and \( \Gamma \) consists of the \textit{a posteriori} scale factors for the 7 source types.

\( \Gamma_1 \) is the scaling factor for \textit{dirA};

\( \Gamma_2 \) is the scaling factor for \textit{indA};

\( \Gamma_3 \) is the scaling factor for \textit{waste};

\( \Gamma_4 \) is the scaling factor for \textit{industry};

\( \Gamma_5 \) is the scaling factor for \textit{energy};

\( \Gamma_6 \) is the scaling factor for \textit{natsoil};

\( \Gamma_7 \) is the scaling factor for \textit{BB}.

Since the Bayesian inversion is conducted each month for each inversion, there are 24×30=720 hourly observed and simulated mixing ratios. The dimension of the matrix \( K \) is 720×7 . The cost function \( J(\Gamma) \) used to determine \( \Gamma \) is Equation 2. The \textit{maximum a posteriori} (MAP) solution of \( \Gamma \) is to minimize the cost function \( J(\Gamma) \), \( S_\epsilon \) is the observational error covariance matrix, the matrix equals \( \sigma_0 \cdot I \), where \( I \) is the identity matrix. We are assuming the observational error (\( \sigma_0 \)) is constructed and calculated via quadrative, \( \sigma_\epsilon = \sqrt{\sigma_m^2 + \sigma_p^2 + \sigma_{mh}^2 + \sigma_b^2} \), where \( \sigma_m \) is the measurement error, \( \sigma_p \) is the error from particles, \( \sigma_{mh} \) is the error from simulated mixing height, \( \sigma_b \) is the error from background mixing ratios, in a unit of ppb. For simplicity, these individual errors are independent.

\( S_\sigma \) is the \textit{a priori} error covariance matrix, it’s a diagonal matrix. Along the diagonal, the percentage uncertainty is given corresponding to \textit{the a priori} uncertainty of each source type.